It's true that Greek mathematicians knew a lot about conics (Apollonius) and few curves given by various constructions that we would say were cubics or quartics, but they didn’t think of them that way, and of these a few had singular points, but that too was not part of their way treating them. You surely don’t want to go that far back.

The early history of the projective geometry of curves is devoted to a remarkable degree to the study of conic sections. There are some related ideas of Greek origin (Pappus’ theorem) and of renaissance origin, but the big mathematical influence came from Desargues and Pascal. However, Desargues’s best writings were as good as lost until Chasles found a copy of his main work in 1845, and Pascal’s study of what he called the hexagrammicum mysticum survives only in part thanks to some notes by Leibniz (its results were largely rediscovered, so far as one tell, by the 1820s and concern the ‘mother theorem’ from which all other theorems about conics appear as consequences). This and related work addressed such questions as giving a unified treatment of all conic sections rather than separate accounts of ellipses, parabolas, and hyperbolas, and seeing them as plane curves rather than sections of a cone (or exploiting the relationship between these ideas).

The subject then passed to Newton, who published a neat construction for generating curves (often singular curves, but he needed it to generate conics) in his *Principia*. There is also his classification of cubic curves into five projective classes and 67 affine classes (although a birational transformation is used at one point). Thereafter there is work on cubics by MacLaurin, who showed, for example, that the flexes lie in threes on straight lines, Braikenridge, Cramer, and Euler. Some of this is in MacaulayBook. All this relies heavily of Cartesian coordinate methods, apart from some remarks by Newton about conics.

There was a huge surge of interest in the study of conics in the years after 1800. What makes it of lasting importance is that it was the place where ideas about duality (via pole and polar), lines at infinity (asymptotes of hyperbolas), letting points on a conic coincide so the line through them becomes a tangent, projective transformations, cross-ratio and its invariance, how metrical properties and projective ones relate, how to find the (or all) conics through n points and tangent to 5-n lines were all worked out.

The over-simplified view of these developments is that it started with the work of Monge on the use of plan and elevation to enable the study of figures in space by their projections onto two planes. Monge was a hugely influential teacher on either side of the French Revolution. Around him were people like Brianchon (who discovered the dual of Pascal’s theorem about six points on a conic, so six tangents to a conic). Also, and famously Poncelet, who published his remarkable *Traité des Propriétés Projective des Figures* in 1822. He shared with Brianchon the desire to have a form of geometrical reasoning that did not break up into special cases (the foot is this perpendicular does or does not lie in this segment, etc.) and so could let geometry to the generality of algebra. This led him into a language in which any two conics could be projectively transformed to two circles (on his definition of common chords and intersection points). Most historians and later mathematicians forgive him because of the wonders of the Poncelet closure theorem, which has had a vigorous afterlife. He was followed by Chasles, who did much to calm the subject down and develop it systematically using the idea of cross-ratio and its invariance. Steiner did much the same in Germany, giving the first good projective definitions of a conic. (The first complete elimination of metrical concepts from projective geometry had to wait for Klein.)

The old historical idea was that these guys advocated synthetic methods in opposition to analytical (coordinate) methods, on the grounds that it was more fundamental and easier. Fundamental because after all a line segment is what it is before you use metrical and coordinate concepts to handle it, and easier because once you learn the right new techniques they can help you find results that algebra makes too complicated to be found. More recent historical work finds that geometers of the time were willing to use a mixture of methods, and generally whatever came to hand, despite their self-identification with one or another programme.

Poncelet was opposed by Gergonne, the capricious editor of what is often said to be the first journal devoted exclusively to mathematics. In his view, algebraic methods could also be upgraded to match the synthetic ones, as he endeavoured to show, and so the distinction was bogus. He engaged in a polemic with Poncelet about duality, which Gergonne saw as a fundamental new idea and Poncelet saw (mathematically correctly) as the same as pole—polar duality in the plane. Gergonne also pushed for the study of the duality of higher plane curves (cubics, etc.). That’s where the duality paradox comes in: the dual of the dual of a curve should be the original curve, but a naïve calculation of its degree says that that cannot be true. And this is where Plücker and his study of the double points, simple cusps, bitangents and flexes took off. He largely confined his interest to cubic and quartic curves -- a step beyond conics that synthetic geometers found almost too difficult to take – but still treated the line at infinity as a special line (no suggestion that a projective transformation could send it into the finite plane). Möbius and later Hesse used homogeneous coordinates to achieve a more systematic and general theory.

Whether this was the study of real curves or complex ones is hard to say. With Plücker it was largely real, but the more algebraic it became the more ambiguous it became. One might have a real curve, or even two in view, and the algebra tells you one thing but the real plane geometry another, so at certain intersection points of one curve with another the curves acquired complex coordinates, but there was no attempt to study all algebraic equations that way. As late as 1873 Cayley expressed himself surprised that there seemed to be no study of complex loci as subsets of $\C \times \C$, although the study of real curves as subsets of $\R \times \R$ had been standard for a long time.

Which is why I was reluctant to say that $\CP^2$ was in mathematical hands in the 1820s.